Dhaka Central University

স্পেশাল ম্যারাথন ক্লাস

উচ্চতর গণিত ১ম পত্র

Chapter: অন্তরীকণ,যোগজীকরণ

❖ $\sqrt{x}+\sqrt{y} = \ln e$ হলে, dy/dx=?

A) $-\sqrt{\frac{y}{x}}$ B) $-\sqrt{\frac{x}{y}}$ C) $\sqrt{\frac{x}{y}}$ D) $\sqrt{\frac{y}{x}}$

$\sqrt{x}+\sqrt{y} = \ln e$

$\Rightarrow \frac{d}{dx}(\sqrt{x}+\sqrt{y}) = \frac{d}{dx}\ln e$

$\Rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \times 2\sqrt{y}$

$= -\frac{\sqrt{y}}{\sqrt{x}}$

$= -\sqrt{\frac{y}{x}}$

$$\lim_{x \to \infty} \frac{x^2 + 6x}{2x^2 + 5} = ?$$

$\frac{f(x)}{g(x)}$

$= 1/2$

A) 0 B) 3/2 C) ½ D) 1

$\lim_{x \to \infty} \frac{f(x)}{g(x)}$

$\frac{1}{2}$

x এর কোন মানের জন্য $y = x + \frac{1}{x}$ এর ঢাল শূন্য হবে?

A) +-2 B) +-1 C)1 D) +-3

$\frac{dy}{dx}$ = ঢাল

$y = x + \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$

যেহু $\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{1}{x^2} = 0$

$x = \pm 1$

$$\lim_{x \to 0} \frac{\sin 2x}{2x^2 + x} = ?$$

$x = 0 \left(\frac{0}{0}\right)$ L'Hospital

A) 1 B) 2 C) ½ D) -2

$$\lim_{x \to 0} \frac{\cos 2x \cdot 2}{4x + 1} =$$

$x = 0$

$$\frac{\cos 0 \cdot 2}{4 \cdot 0 + 1} = \frac{2}{1} = 2$$

$$\lim_{x \to 0} \frac{\sin 7x - \sin x}{\sin 6x} = ?$$

A) 7/6 B) -7/6 C) 1 D) 0

$\frac{7-1}{6} = \frac{6}{6} = 1$

$\lim_{x \to 0} \frac{\sin ax - \sin bx}{\sin cx}$

$= \frac{a-b}{c}$

$$\lim_{x \to 0} \frac{7 \sin\left(\frac{x}{7}\right)}{x} = ?$$

$\frac{x}{7}$

A) 0 B) 1/7 C) 1 D) 7

$$\lim_{x \to 0} \frac{7 \sin\left(\frac{x}{7}\right) \cdot \frac{1}{7}}{\frac{x}{7}}$$

$= 1$

$$\lim_{x \to 0} \frac{\sin ax}{ax} = 1$$

$$\frac{d}{dx}(\log_x e) = ?$$

$$\frac{d}{dx}\left(\frac{1}{\ln x}\right) = \frac{d}{dx}(\ln x)^{-1}$$

$$= -1 \cdot (\ln x)^{-2} \cdot \frac{1}{x}$$

$$= -\frac{1}{x(\ln x)^2}$$

Ans

$$\log_x e = \frac{1}{\log_e x}$$

$$= \frac{1}{\ln x}$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

এর মান কত?

A) –e B) 1/e C)2e D) e

❖ $x<0$ এর কোন মানের জন্য $f(x)=x+1/x$ ফাংশনটির সর্বোচ্চ মান সম্পন্ন হবে?

A) -5 B) -1 C) 0 D) 2

$f'(x)=0$

$\Rightarrow 1-\frac{1}{x^2}=0$

$\Rightarrow \frac{1}{x^2}=1$

$x=\pm 1$

$x<0$

$x=-1$

$$\lim_{x \to \infty} \frac{x^2 + 6x}{2x^2 + 5}$$ এর মান কত?

A) 3/2 B) ½ C) 1 D) 0

$$\lim_{x \to \infty} \frac{2x}{(4x+1)} = ?$$

A) 0 B) 2 C) ½ D) 0

$y=\sqrt{x}$ বক্ররেখার উপর $x=4$ বিন্দুতে স্পর্শকের ঢাল? $\frac{dy}{dx}$

A) 1 B) ½ C) 1/3 D) 1/4

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$$

$x=4$ হলে $\frac{dy}{dx}=\frac{1}{2\cdot\sqrt{4}}=\frac{1}{2\cdot 2}=\frac{1}{4}$

❖ y=ln(2-x) হলে dy/dx =?

A)1/(2-x) B) 2/(2-x) C) x/(2-x) D) 1/(x-2)

$$\frac{dy}{dx} = \frac{1}{(2-x)}(-1)$$

$$= \frac{1}{(x-2)}$$

❖ যদি $y=kx(2x+\sqrt{3})$ বক্ররেখার মূলবিন্দুতে স্পর্শকটি অক্ষের সাথে $30°$ কোণ উৎপন্ন করে তাহলে k এর মান হবে–

(0,0)

A) √3 B) 1/√3 C) 1/3 D) 1

$\frac{dy}{dx} = \tan 30°$

$y = 2kx^2 + kx\sqrt{3}$

$\frac{dy}{dx} = 4kx + k\sqrt{3}$

(0,0) বিন্দুতে $\frac{dy}{dx} = k\sqrt{3}$

$\therefore k\sqrt{3} = \tan 30°$

$\therefore k\sqrt{3} = \frac{1}{\sqrt{3}}$

$k = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$

$\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 2x}$ এর মান হবে- $\frac{0}{0}$

$\frac{1}{4}$

$\lim_{x \to 0} \frac{0 - (-\sin x)}{2\sin 2x \cdot \cos 2x \cdot 2}$

$\lim_{x \to 0} \frac{\sin x}{2 \cdot \sin x \cdot \cos x \cdot 2}$

$\lim_{x \to 0} \frac{1}{4\cos x}$ | $x \to 0 \Rightarrow \frac{1}{4}$

DCU Final Model Test

$$\lim_{x \to \infty} 2^x \sin\left(\frac{y}{2^x}\right) = ?$$

y

❖ $e^{(xy+1)} = 5$ হলে dy/dx কোনটি?

Ans $-\frac{y}{x}$ ✗

$\ln(e^{xy+1}) = \ln 5$

$\Rightarrow (xy+1)\ln e = \ln 5$

$\Rightarrow \frac{d}{dx}(xy+1) = \frac{d}{dx}\ln 5$

$\Rightarrow y + x\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = -\frac{y}{x}$

$$\lim_{x \to 0} \frac{1 - \cos 2x}{x} = ?$$

$\frac{0}{0}$

A) 0 B) 1 C) -1 D) ∞

L' Hopital

$$\lim_{x \to 0} \frac{\sin 2x \cdot 2}{1} = 0$$

❖ x এর কোন মানের জন্য y=xlnx এর লঘু মান নির্ণয় করা যাবে?

A) e B) ln e C) 1/e D)-e

$$\frac{dy}{dx} = \ln x + x\frac{1}{x}$$

$$= \ln x + 1$$

লঘুমান শর্ত

$$\ln x + 1 = 0$$

$$\ln x = -1$$

$$x = e^{-1} \quad \left(\frac{1}{e}\right)$$

$e^y=\tan^{-1}x$ হলে $dx/dy=?$

$$e^y = \tan^{-1}x$$

$$y = \ln(\tan^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\tan^{-1}x} \cdot \frac{1}{1+x^2}$$

$$\therefore \frac{dx}{dy} = (1+x^2)(\tan^{-1}x)$$

যদি $y = \ln(\ln x)$ হয়,তবে $\frac{dy}{dx}$ এর মান কোনটি?

$$\frac{dy}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$= \frac{1}{x \ln x}$$ ✓

$$\int \frac{x\,dx}{\sqrt{1-x^2}} = ?$$

$\frac{d}{dx}(1-x^2) = -2x$

$$-\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$$= -\frac{1}{2}\,2\sqrt{1-x^2} + c$$

$$= -\sqrt{1-x^2} + c$$

Short cut

$$\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$$

$$\int_{-1}^{1} |x|dx = ?$$

A) 2 B) -1 C) 1 D) 0

$$\int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx$$

$$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= 1+1 = 2$$

$|x| \longrightarrow x > 0$

$\longrightarrow -x < 0$

$-1 \to 0$

$0 \to 1$

$$\int \frac{2\tan^{-1}x}{1+x^2}dx = f(x) + c$$ হলে f(x)=?

$\int 2z\,dz = 2.\frac{z^2}{2} + c$

$= z^2 + c$

$= (\tan^{-1}x)^2 + c$

$f(x) + c$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

$\tan^{-1}x = z$

$\frac{1}{1+x^2}dx = dz$

❖ $y^2=4x$ এবং $y = x$ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

A) 8 B) 8/3 C) 3/8 D) 4

$y^2 = 4ax$ এবং $y = mx$

এদের দ্বারা আবদ্ধ ক্ষেত্রফল $= \frac{8a^2}{3m^3}$

$y^2 = 4.1.x$

$a = 1$

$y = 1.x$

$\therefore m = 1$

ক্ষেত্রফল $= \frac{8}{3}$

$$\int_0^4 \sqrt{16 - x^2}\,dx = ?$$

A) 2/3 B) -4π C) -1 D) 4π

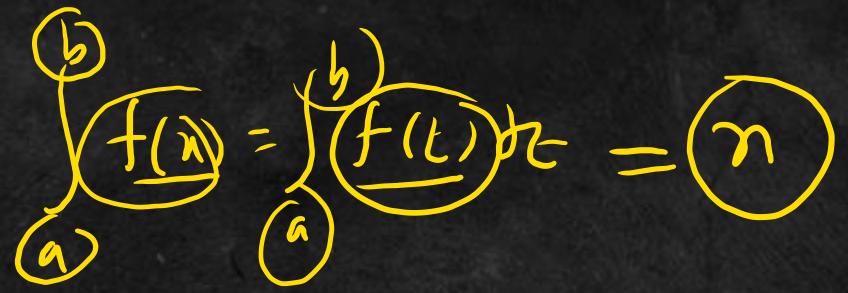
$= \frac{\pi}{4} \times 16$

$= 4\pi$

$\int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi}{4} \times a^2$

$$\int_0^4 f(x)dx = 6, \int_{-1}^{3} f(x+1)dx = ?$$

A) 6 B) 5 C) 7 D) 0



$$\int_{-1+1}^{3+1} f(t)\,dt = \int_0^4 f(t)\,dt = \int_0^4 f(x)\,dx = 6$$

$$\int \frac{dx}{e^x + e^{-x}} = ? + C$$

$$\int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{e^x}{1+(e^x)^2} dx$$

$$\int \frac{1}{1+u^2} du = \tan^{-1} u$$

$$= \tan^{-1}(e^x) + c$$

$\int_0^{\frac{\pi}{2}} \cos^3 x dx$ এর মান কত?

$$\frac{1}{4}\int_0^{\pi/2} 4\cos^3 dx = \frac{1}{4}\int_0^{\pi/2} (\cos 3x + 3\cos x)\,dx$$

$$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^{\pi/2}$$

$$= \frac{1}{4}\left[\frac{\sin \frac{3\pi}{2}}{3} + 3\sin\frac{\pi}{2} - 0\right]$$

$$= \frac{1}{4}\left[\frac{-1}{3} + 3\right]$$

$$= \frac{1}{4} \cdot \frac{8}{3} = \frac{2}{3}$$

❖ $y=\sin x$, $y=0$, $x=0$ এবং $x=\pi$ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল কত?

$$\int_0^{\pi} y\,dx = \int_0^{\pi} \sin x\,dx$$

$$= \left[-\cos x\right]_0^{\pi}$$

$$= -\cos\pi + \cos 0^\circ$$

$$= 1+1 = 2$$

$$\int_1^e \ln x\,dx = ?$$

$\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$

$\ln x = u$

$v = 1$

$$\int_1^e \ln x\,dx = \ln x \int 1\,dx - \int\left(\frac{d}{dx}\ln x \cdot \int 1\,dx\right)dx$$

$$= \ln x \cdot x - \int\left(\frac{1}{x}\cdot x\right)dx$$

$$= [x\ln x - x]_1^e$$

$$= [e\ln e - e - 1\ln 1 - 1]$$

$$= 1$$

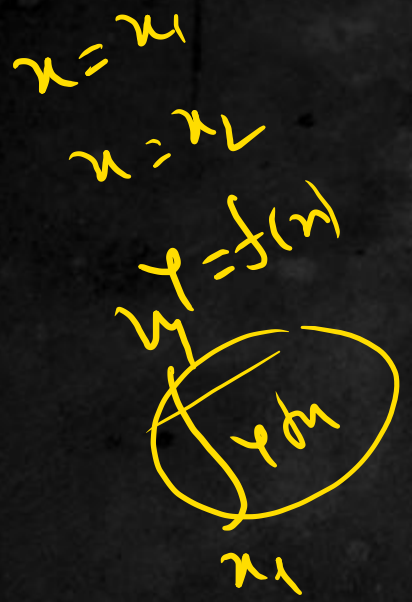
$$\int_0^1 \frac{x}{\sqrt{1-x^2}}dx = ?$$

$$= \left[-\sqrt{1-x^2}\right]_0^1$$

$$= \left(-\sqrt{1-1} + \sqrt{1-0}\right)$$

$$= 1$$

❖ y=2x. x অক্ষ এবং x=2 লাইনসমূহ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল?



$x=0,\ x=2$

$$\int_0^2 2x\,dx = \left[2\cdot\frac{x^2}{2}\right]_0^2$$

$$= [x^2]_0^2$$

$$= 4$$

$\int_0^1 \frac{x}{2-x^2} dx$ এর মান কত?

$$-\frac{1}{2}\int_0^1 \frac{-2x}{2-x^2}\,dx$$

$$\Rightarrow \left[-\frac{1}{2}\ln(2-x^2)\right]_0^1$$

$$= -\frac{1}{2}\left[\ln(2-x^2)\right]_0^1$$

$$= -\frac{1}{2}(\ln 1 - \ln 2)$$

$$\Rightarrow -\frac{1}{2}\ln\frac{1}{2}$$ ✓

$$\int 2\sin^2\left(\frac{x}{2}\right)dx = \text{কত?}$$

$2\sin^2\theta = 1-\cos 2\theta$

$$\int\left(1-\cos 2\cdot\frac{x}{2}\right)dx$$

$$= \int(1-\cos x)dx$$

$$= (x-\sin x)+c$$

❖ y=0, x=1 এবং y=x রেখা তিনটি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল-

y=0
x=0

T

Ideal

$\int_0^1 y\,dx \Rightarrow \int_0^1 x\,dx$

$= \left|\frac{x^2}{2}\right|_0^1$

$= \frac{1}{2}$

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